USPAS FEL 2021 Homework Set 1

1.1 The Argonne Photon Source (APS) has the following electron beam properties:

E_b = 7 GeV

single bunch charge = 600 pC

rms bunch length = 30 ps

geometric emittance in x ~ 8 nm-rad

geometric emittance in y ~ 0.3 nm-rad

a. Calculate the beam's relativistic factor γ
b. Calculate the normalized emittance in x and y
c. Calculate the number of electrons in each bunch
d. Consider a "typical" undulator with λ_u = 3 cm, K = 1.32 and N_u (number of periods) = 100. Calculate the undulator radiation fundamental wavelength.
e. To a good approximation, the x-ray flux, i.e., number of x-ray photons emitted per second over the $1/N_u$ spectral bandwidth, for undulator radiation is given by

$$N_p = \pi \alpha N_e \frac{K^2}{\left(1 + \frac{K^2}{2}\right)}$$

where α = 1/137. Calculate the number of x-ray photons emitted in each 30-ps pulse.

f. Calculate the APS brightness at the fundamental wavelength.

1.2 The LCLS CuRF linac has the following nominal electron beam properties:

E_b = 10 GeV
single bunch charge = 160 pC
electron bunch FWHM ~ 50 fs
normalized emittance in both x and y ~ 0.4 μm-rad
rms energy spread of 2 MeV

a. Calculate the beam's relativistic factor γ
b. Calculate the peak current
c. Calculate the relative energy spread
d. Calculate the electron beam peak brightness

1.3 The new hard x-ray (HXR) undulator has a 26-mm period and adjustable gap.

a. If the gap is set such that the undulator K = 2, calculate the x-ray wavelength and photon energy generated by the electron beam specified in Problem 1.2.
b. Calculate the maximum transverse position of the electron beam in the undulator
c. Calculate the maximum transverse velocity of the electron beam in the undulator
d. If the FEL saturated power is 30 GW, calculate the number of x-ray photons in each pulse
e. Calculate the peak brightness, assuming all photons are emitted within a 0.1% relative bandwidth.
f. Calculate the enhancement factor over the APS brightness calculated in problem 1.1.

1.4 In the problem, use the expressions for the Gaussian pulse (slide 20) and Gaussian laser beam (slide 24) to confirm the following equalities.

a. Derive the following equality between the FWHM and the rms temporal width below

$$\delta t = 2\sqrt{2ln2}\,\sigma_t$$

b. Derive the following equality between the Gaussian laser beam and the 1/e2 radius.

$$\delta r_{FWHM} = \sqrt{2ln2}\,w_0$$